Factors affecting the valuation of corporate bonds

Edwin J. Elton [a,*], Martin J. Gruber [a],
Deepak Agrawal [b], Christopher Mann [c]

[a] *Stern School of Business, 44 West 4th Street, New York, NY 10012, USA*
[b] *KM Moody's, 1620 Montgomery Street, San Francisco, CA 94111, USA*
[c] *Moody's, 99 Church Street, New York, NY 10007, USA*

Available online 14 August 2004

Abstract

An important body of literature in Financial Economics accepts bond ratings as a sufficient metric for determining homogeneous groups of bonds for estimating either risk-neutral probabilities or spot rate curves for valuing corporate bonds. In this paper we examine Moody's and Standard & Poors ratings of corporate bonds and show they are not sufficient metrics for determining spot rate curves and pricing relationships. We investigate several bond characteristics that have been hypothesized as affecting bond prices and show that from among this set of measures default risk, liquidity, tax liability, recovery rate and bond age leads to better estimates of spot curves and for pricing bonds. This has implications for what factors affect corporate bond prices as well as valuing individual bonds.
© 2004 Elsevier B.V. All rights reserved.

---

* Corresponding author. Address: Management Education Center, Stern School of Business, 44 West 4th Street, New York, NY 10012, USA.

E-mail addresses: eelton@stern.nyu.edu (E.J. Elton), mgruber@stern.nyu.edu (M.J. Gruber), deepak.agrawal@moodys.com (D. Agrawal), christopher.mann@moodys.com (C. Mann).

1. Introduction

The valuation of corporate debt is an important issue in asset pricing. While there has been an enormous amount of theoretical modeling of corporate bond prices, there has been relatively little empirical testing of these models. [1] Recently there has been extensive development of rating based reduced form models. These models take as a premise that bonds when grouped by ratings are homogeneous with respect to risk. For each risk group the models require estimates of several characteristics such as the spot yield curve, the default probabilities and the recovery rate. These estimates are then used to compute the theoretical price for each bond in the group. The purpose of this article is to examine the pricing of corporate bonds when bonds are grouped by ratings, and to investigate the ability of characteristics, in addition to bond ratings, to improve the performance of models which determine the theoretical prices. While a number of authors have used bond ratings as the sole determinant of quality, implicit or explicit in much of this work is the idea that a finer classification would be desirable. [2] This is the first paper to explicitly test both one at a time and simultaneously the impact of a large set of additional variables on bond prices across a large sample of corporate bonds. Most of our testing will be conducted in models which are in the spirit of the theory developed by Duffie and Singleton (1997) and Duffee (1999).

The paper is divided into three sections. In the first section, we briefly discuss reduced form models that have been suggested in the literature. In the second section we examine how well standard classifications serve as a metric for forming homogeneous groups. In that section we show that using standard classifications results in errors being systematically related to specific bond characteristics. Finally, in the last section we take account of these specific bond characteristics in our estimation procedure for determining spot prices and show how this lead to improved estimates of corporate bond prices.

2. Alternative models

There are two basic approaches to the pricing of risky debt: reduced form models and models based on option pricing. Reduced form models are found in Elton et al. (2001), Duffie and Singleton (1997), Duffee (1999), Jarrow et al. (1997), Lando (1997), Das and Tufano (1996). Option-based models are found in Merton (1974)

---

[1] Most testing of theoretical models has been performed using other types of debt. Cumby and Evans (1997) examine Brady bonds, Merrick (1999) examines Russian bonds and Madan and Unal (1998) examine Certificates of Deposit.

[2] Examples of studies using an agencies rating to define homogenous risk classes include Jaffee (1975), Boardman and McEnally (1981), and Elton et al. (2001). Attempts to incorporate additional variable include issue size, e.g., Lamy and Thompson (1988), differences between Moody's and Standard and Poor ratings, e.g., Perry et al. (1988) and Billingsley et al. (1985), and a number of market makers and security issuer type, e.g., Bradley (1991).

and Jones et al. (1984). In this paper we will deal with a subset of reduced form models, those that are rating based. Discussion of the efficacy of the second approach can be found in Jones et al. (1984).

The basic structure of reduced form models follows from the existence of a risk-neutral measure and the absence of arbitrage. It follows from this that the value of a bond is the certainty equivalent cash flows (at risk-neutral probabilities) brought back at risk free rates. For a two-period bond that has a face value of $1, value can be expressed as follows:

$$\text{Value}_0 = \frac{C(1 - \lambda_1) + a\lambda_1}{(1 + r_1)} + \frac{(C + 1)(1 - \lambda_1)(1 - \lambda_2) + a\lambda_2(1 - \lambda_1)}{(1 + r_2)^2}, \tag{1}$$

where:

(1) C is the coupon,
(2) a is the recovery rate,
(3) r_t is the riskless rate from 0 to t,
(4) λ_j are the term structure of risk-neutral probabilities of default at time 0 for all periods $j = 1, \ldots, J$.

The issue is how to estimate the risk-neutral probabilities. Risk-neutral probabilities are either estimated for an individual firm using the bonds the firm has outstanding or for a group of firms that are believed to be homogeneous. [3]

The use of a homogeneous risk class has the advantage of a much larger sample size for estimating risk-neutral probabilities. However, this approach leaves us with the problem of defining homogeneous risk classes. In this paper we will explore how to determine a homogeneous group to minimize risk differences. Like Jarrow et al. (1997), we will initially assume that Moody's or S&P rating classes are a sufficient metric for defining a homogeneous group. We will then show that pricing errors within a group vary with bond characteristics. How these variations can be dealt with and the improvement that comes from accounting for these differences will then be explored. We will do so using a form of the Duffie and Singleton (1997) model to price corporate bonds. The great strength of this approach is that with this model using risk-neutral probabilities and riskless rates is equivalent to discounting promised cash flows at corporate spot rates. Thus the problem of estimating corporate bond prices can be reduced to the problem of estimating corporate spot rates.

3. Analysis based on rating class

In this section we initially accept Moody's rating as a sufficient metric for homogeneity and investigate the pricing of bonds under this assumption. We start by

---

[3] Another possibility and one that should be explored in the future is to extract risk-neutral probabilities from the price of credit default swaps. These have greater liquidity than bonds.

describing our sample and the method used to extract spot rates for corporate bonds. We then examine the pricing errors for bonds when this technique is applied.

3.1. Data

Our bond data is extracted from the Lehman Brothers Fixed Income database distributed by Warga (1998). This database contains monthly price, accrued interest, and return data on all investment grade corporate bonds. In addition, the database contains descriptive data on bonds including coupon, ratings, and callability.

A subset of the data in the Warga database is used in this study. First, any bond that is matrix-priced rather than trader-priced in a particular month is eliminated from the sample for that month. Employing matrix prices might mean that all our analysis uncovers is the formula used to matrix-price bonds rather than the economic influences at work in the market. Eliminating matrix-priced bonds leaves us with a set of prices based on dealer quotes.

Next, we eliminate all bonds with special features that would result in their being priced differently. This means we eliminate all bonds with options (e.g., callable or sinking fund), all corporate floating rate debt, bonds with an odd frequency of coupon payments, government flower bonds and index-linked bonds. [4] Next, we eliminate all bonds not included in the Lehman Brothers bond indexes because researchers in charge of the database at Shearson–Lehman indicated that the care in preparing the data was much less for bonds not included in their indexes. Finally we eliminate bonds where the data is problematic. [5] For classifying bonds we use Moody's ratings. In the few cases where Moody's ratings do not exist, we classify using the parallel S&P rating.

Our final sample covered the 10-year period: 1987–1996. Details on sample size are presented in the accompanying tables. The basic sample varied from an average of 42 bonds for the industrial Aa category to 278 bonds for the financial A category.

3.2. Extracting spot rates

In this section we discuss the methods of extracting spots from bond prices and apply it to our sample when Moody's ratings are used to define a homogeneous risk class. [6]

Calculating model prices following Duffie and Singleton involves the discounting of promised cash flows at spot rates. Implementing this procedure is straightforward. First, spot rates must be estimated. In order to find spot rates, we used the Nelson

---

[4] The alternative was to construct a model which explicitly prices the option like features. While this is an interesting project, it is helpful to understand the determination of risk and homogeneity before dealing with option pricing.

[5] Slightly less than 3% of the sample was eliminated because of problematic data. The eliminated bonds had either a price that was clearly out of line with surrounding prices (pricing error) or involved a company or bond undergoing a major change.

[6] For an application of this methodology applied to government bonds, see Elton et al. (2001).

Table 1
Pricing errors based on rating classes

	Financial sector			Industrial sector		
	Aa	A	Baa	Aa	A	Baa
Average pricing errors	0.0094	0.0104	−0.0149	−0.0162	−0.0082	0.0094
Average absolute pricing errors	0.335	0.593	0.884	0.475	0.625	1.172

This table shows the average pricing errors when promised payments are discounted at the corporate rates. Discounted rates on promised payments were fitted each month separately for each rating category of bonds. Errors are the fitted prices minus the invoice prices of coupon bonds. Errors are expressed in dollars on bonds with face value of 100 dollars.

and Siegel (1987) procedure for estimating spots from a set of coupon paying bonds. For each rating category, including governments, spots can be estimated at a point in time (zero) as follows: [7]

$$P_{i0} = \sum_{t=1}^{T} D_t CF_{it},$$

$$D_t = e^{-r_{0t}t},$$

$$r_{0t} = a_0 + (a_1 + a_2) \left[\frac{1 - e^{-a_3 t}}{a_3 t} \right] - a_2 e^{-a_3 t}, \tag{2}$$

where P_{i0} is the price of bond i at time 0; CF_{it} is the promised cash flow on bond i that is expected to occur t periods later; D_t is the present value as of time zero for a payment that is received t periods in the future; r_{0t} is the spot rate at time zero for a payment to be received at time t; a_0, a_1, a_2, and a_3 are parameters of the model.

For each rating class for each month, these equations are fitted to the cash flows for all bonds in that rating class to minimize mean squared pricing error. Discounting the promised cash flows on each bond in a particular rating class at the estimated spot rates for that rating class produces the model price for that bond. Table 1 presents the pricing errors when this technique is used. For all rating classes the average pricing error is close to zero. The average error is less than 1 cent per $100 of the face value of the bond over the sample period. The Nelson–Siegel procedure, like all curve-fitting techniques, pre-specifies a functional form for the discount rates. If an inappropriate functional form is chosen, pricing errors might be a function of maturity while producing average errors close to zero. To analyze this we computed

---

[7] See Nelson and Siegel (1987). For comparisons with other procedures, see Green and Odegaard (1997) and Dahlquist and Svensson (1996). We also investigated the cubic spline procedures and found substantially similar results throughout our analysis. The Nelson and Siegel model was fit using standard Gauss–Newton non-linear least squared methods. The Nelson and Siegel (1987) and cubic spline procedures have the advantage of using all bonds outstanding within any rating class in the estimation procedure, therefore lessening the effect of sparse data over some maturities and lessening the effect of pricing errors on one or more bonds. The cost of these procedures is that they place constraints on the shape of the yield curve.

average differences between model prices and dealer prices (errors) at each interval of maturity used in Table 2. The average pricing error across maturities varied from 0.1 cent per $100 to 2.6 cents per $100. This is very similar to the overall average pricing error across all maturities of one cent per $100. Furthermore, the errors showed no pattern across maturities. No part of the maturity spectrum was systematically over- or underpriced. This is consistent with what others have found when using the Nelson and Siegel procedure (see Green and Odegaard, 1997; Dahlquist and Svensson, 1996). Thus the pre-specified functional form in Nelson and Siegel seems to be general enough not to introduce systematic pricing errors.

We can learn more about risk classes by examining the absolute pricing errors produced by the Nelson–Siegel procedure. This is a measure of the dispersion of errors across bonds within one rating class and thus, of how homogeneous the rating class is. The results in Table 1 show that while Moody's rating classes do an excellent job of pricing the ''average bond'' there are large errors in pricing individual bonds. The errors vary from 34 cents per $100 for Financials Aa's to over $1.17 for Baa industrials. This suggests that there are other variables that systematically effect bond prices and by studying pricing errors we can uncover the additional influences. In the next section we will explore this issue.

4. Other factors that affect risk

When estimating spot rates, one has to make a decision as to how to construct a group of bonds that is homogeneous with respect to risk. In the prior section we followed other researchers by accepting the major classifications of rating agencies. In this section we explore the use of additional data to form more meaningful groups.

In general, when dividing bonds into subsets, one faces a difficult tradeoff. The more subsets one has, the less bonds are present in any subset. Bond prices are subject to idiosyncratic noise as well as systematic influences. The more bonds in a subset, the more the idiosyncratic noise is averaged out. This suggests larger groupings. However, if the subset is not homogeneous, one may be averaging out important differences in underlying risk and misestimating spot rates because they are estimated for a group of bonds where subsets of the group have different yield curves.

What are the characteristics of bonds that vary within a rating class that could lead to price differences? We will examine the following possibilities:

(A) default risk,
(B) liquidity,
(C) tax liability,
(D) recovery rates,
(E) age.

In this section, we examine influences one at a time. This is useful for understanding the magnitude and structure of the relationships. In later sections, we will examine simultaneously the influences which prove important.

Table 2
Model errors due to maturity and gradations within ratings industrial sector (Panel A) and financial sector (Panel B)

	1.0–2.0	2.01–4.0	4.01–6.0	6.01–8.0	8.01–10.0	10.01–10.99	Overall
Panel A							
Aa							
Number of bonds							
+	34	130	129	108	172	18	591
0	360	634	509	365	398	62	2328
−	228	452	448	502	559	75	2264
Average error							
+	0.112	−0.152	0.360	0.255	0.517	−0.113	0.245
0	0.045	−0.015	0.004	0.065	0.009	−0.216	0.010
−	0.084	0.030	0.061	−0.095	−0.227	0.378	−0.038
	(0.786)	(2.564)	(6.474)	(5.872)	(13.502)	(2.559)	(9.804)
A							
Number of bonds							
+	707	1364	1425	1176	1173	178	6023
0	752	1549	1692	1423	1641	200	7257
−	511	1092	1423	1481	1613	275	6395
Average error							
+	0.171	0.288	0.504	0.524	0.622	0.531	0.443
0	−0.005	−0.111	−0.078	−0.145	−0.133	0.160	−0.096
−	−0.095	−0.237	−0.225	−0.279	−0.391	−0.355	−0.277
	(14.406)	(22.126)	(27.458)	(23.091)	(27.128)	(7.459)	(50.607)
Baa							
Number of bonds							
+	361	866	889	864	1257	66	4303
0	324	938	1068	965	1255	149	4699
−	393	1037	1039	1094	1236	93	4892
Average error							
+	0.374	0.684	0.932	0.839	1.009	1.415	0.846
0	0.242	0.039	0.116	0.266	0.278	0.500	0.196
−	−0.391	−0.567	−0.662	−1.013	−1.287	−1.509	−0.873
	(16.732)	(25.938)	(28.186)	(29.286)	(36.386)	(10.445)	(61.207)
Panel B							
Aa							
Number of bonds							
+	218	207	36	47	44	0	552
0	306	616	642	420	294	12	2290
−	1284	2081	1283	705	551	44	5948
Average error							
+	−0.044	−0.055	−0.131	−0.283	−0.369	–	−0.100
0	−0.049	0.014	−0.066	−0.055	0.046	−0.707	−0.029
−	−0.025	0.056	−0.062	−0.024	0.166	0.064	0.014
	(1.267)	(4.490)	(0.786)	(3.334)	(4.965)	(0.214)	(6.458)

Table 2 (*continued*)

	1.0–2.0	2.01–4.0	4.01–6.0	6.01–8.0	8.01–10.0	10.01–10.99	Overall
A							
Number of bonds							
+	1838	3131	2146	1486	1475	110	10186
0	2100	4014	2604	2134	2378	222	13452
−	903	2112	2352	2352	2168	262	10149
Average error							
+	−0.112	−0.179	−0.491	−0.575	−0.646	−0.288	−0.359
0	−0.065	−0.025	−0.143	−0.127	−0.038	−0.163	−0.075
−	0.163	0.460	0.368	0.417	0.608	0.173	0.426
	(18.292)	(34.381)	(37.323)	(36.805)	(37.658)	(5.852)	(70.620)
Baa							
Number of bonds							
+	843	1562	1092	1157	1499	123	6276
0	333	568	831	758	836	64	3390
−	131	228	254	350	365	4	1332
Average error							
+	−0.168	0.020	−0.255	−0.227	−0.224	−0.128	−0.160
0	0.062	0.118	−0.231	−0.135	0.110	0.142	−0.031
−	0.225	0.349	0.982	0.799	1.036	0.076	0.765
	(4.410)	(4.444)	(13.501)	(11.754)	(14.923)	(0.562)	(22.832)

Moody's rates bonds using broad categories as well as finer gradations (+, 0, and −). Plus securities are designated as less risky than minus securities. This table separates bonds into groups according to these finer gradations (along the left-hand side). It further separates the bonds according to maturity (in years from left to right). The first column represents bonds with maturity between 1.0 and 2.0 years, inclusive. Model price is calculated by discounting promised cash flows at estimated corporate spot rates. Average error is defined as model price minus invoice price. Numbers in parentheses are *t*-statistics to test the null hypothesis that the average errors for the + and − categories are equal.

4.1. Differential default risks

All bonds within a rating class may not be viewed as equally risky. There are several characteristics of bonds which might be useful in dividing bonds within a rating class into new groups. We will examine two of these in this section. We start by examining the subcategories within a rating class which Moody's and Standard & Poors have both introduced. We then examine whether a difference between Moody's and Standard & Poors ratings convey risk information.

We start by examining the finer breakdown of ratings produced by the rating agencies themselves. Standard & Poors and Moody's have introduced plus and minus categories within each letter rating class. One obvious possibility is that bonds that are rated as a plus or a minus are viewed as having different risk than bonds that receive a flat letter rating. If this is true, then estimating one set of spot rates for all bonds in a class should result in consistent pricing errors for bonds rated "plus" (too low a model price and hence negative errors) or bonds rating "minus" (too high a model price and hence positive errors).

Table 2(Panels A and B) explores this possibility. For each rating class the table is split into two sections. The top section shows the number of bond months in each rating class for varying maturity and across all maturities. [8] The bottom section shows the average of the model price minus the invoice price (market price plus accrued interest) for each rating category. For all rating categories, plus-rated bonds have, on average, too low a model price, and minus-rated bonds too high a model price. The difference between the pricing error of plus rated, flat and negative rated bonds is highly statistically significant as shown in Table 2. Furthermore, the differences are of economic significance (e.g., for minus versus flat Baa industrial bonds the overall difference is over 1% of the invoice price). The same pattern is present for most of the maturities with some tendency for the magnitude of errors to increase with the maturity. In addition, the size of the average pricing error increases as rating decreases. Thus, it is most important for Baa bonds. This would suggest that one should estimate a separate spot curve for these subclasses of ratings. However, for much of the sample, the paucity of bonds in many of the subclasses makes it difficult to estimate meaningful spot rates for a subclass. In a latter section we will explore how these differences can be built into an estimation procedure for spot rates. [9]

Bonds might also be viewed as being different in risk if S&P rates the bond differently than Moody's. In Table 3 we explore whether bonds that are given a higher (lower) rating by S&P than by Moody's are considered less (more) risky by investors. Recall that our yield curves are derived using Moody's ratings. The question is whether when Standard & Poors gives the bond a higher rating that Moody's, does an investor believe that the second rating conveys information not contained in the first rating. In considering differences we use pluses and minuses. Thus, if Moody's rates a bond as Baa and S&P rates the bond BBB+, we count this as a difference in ratings. Once again the upper half of the table shows the number of bonds in each category, and the lower half the difference between model price and invoice price. In presenting the data we do not sub-classify by maturity since we found no pattern in pricing errors across maturity.

Investors clearly take the difference in rating into account. If the S&P rating is lower than Moody's, then investors act as if the bond is higher risk than is implied by the Moody's rating and they will set a lower market price, and this results in a model price above invoice price and a positive error. Likewise, if S&P rates the bond higher than Moody's the bond is considered by investors as lower risk compared to bonds where they agree and the pricing error is negative. Almost all of the results are statistically significant at the 1% level. The errors when the rating agencies disagree is statistically different from the errors when they agree. Neither Moody's nor S&P ratings have the dominant influence. When we reversed the table and examined the

---

[8] For all bonds rated by Moody's we use Moody's classification. For the few bonds not rated by Moody's, we use S&P's classification.

[9] We also explored whether the likelihood of a bond being upgraded or downgraded could affect price. The prediction of rating change we used was past rating change. While past rating change has a very weak predictable relationship to future rating change in the same direction, there was no discernable relationship with pricing errors.

Table 3
Model errors due to differences between Moody's and Standard & Poors

	Financial sector			Industrial sector		
	Aa	A	Baa	Aa	A	Baa
Panel A: Number of pricing error observations						
S&P lower	2075	4557	1720	841	4281	3111
S&P same	5198	18 537	3481	2906	9459	6639
S&P higher	1456	10465	5702	1432	5875	4062
Panel B: Average error						
S&P lower	0.015	0.253	0.117	0.080	0.010	0.212
S&P same	−0.020	−0.085	0.009	0.063	0.052	0.000
S&P higher	−0.086	−0.000	−0.066	−0.232	−0.138	−0.237
t-score between lower and higher	1.0351	16.097	5.623	9.995	8.361	12.051

This table examines whether bonds whose S&P rating is different from Moody's rating are viewed by the market as having different risks. Model errors are model price minus invoice price. Units are dollars per $100 bond.

effect when Moody's ratings differed from S&P (rather than S&P from Moody's), we got similar results.

4.2. Different liquidity

The second reason why bonds within a rating class might be valued differently is because they have different liquidity. Data is not available on bid/ask spread, the most direct measure of liquidity, nor is there data on trading volume which is a natural proxy for liquidity. We used three indirect measures of liquidity: dollar value outstanding, the percentage of months a bond was matrix priced, and whether a bond was recently issued. Our logic behind the second measure was that dealers priced the more active issues more often. Thus bonds that were always dealer-priced were likely to be more liquid than bonds that were dealer-priced only part of the time. Neither of the first two measures showed any significant patterns, and so we have not presented a table of results. The third measure rests on the belief that newly issued bonds are more liquid than bonds which have been in the market for a longer period of time. We defined newly issued bonds as bonds that were brought to the market within the previous year. Table 4 shows the difference between newly issued (first-year bonds) and older bonds. Once again the top half is the number of bond months in each cell, and the bottom half is the average difference between model price and invoice price. As shown in Table 4, newly issued bonds sell at a premium compared to model prices and all of these results are highly statistically significant. These results are consistent with newly issued bonds being more liquid. [10]

---

[10] We repeated the analysis matching bond maturity of the newly issued and seasoned bonds. There was no change in results.

Table 4
Model errors due to bond age

	Financial sector			Industrial sector		
	Aa	A	Baa	Aa	A	Baa
Panel A: Number of pricing error observations						
First year	2663	8501	2786	1116	4294	3298
Older	6071	24692	8021	3912	14728	10288
Panel B: Average error						
First year	−0.121	−0.189	−0.059	−0.210	−0.201	−0.268
Older	0.047	0.051	0.009	0.039	0.045	0.058
t-scores between 1st year and older	14.816	21.283	2.338	8.799	14.029	9.522

This table examines the effect of age on pricing errors. The data was separated into two groups: one whose bonds were issued within a year of observation and another whose bonds were older than a year. Panel A gives the number of bonds in each group. Panel B gives the average error for each group. Error is defined as model price minus invoice price. Units are dollars per $100 bond.

4.3. Different tax treatment

The third possible reason why bonds within a risk class might be viewed by investors differently is because they have different after tax value because of the way coupons and capital gains are taxed. Throughout most of the period used in our study the tax rates on capital gains and interest income were the same. However, since capital gains are paid at the time of sale, bonds with lower coupons may be more valuable because some taxes are postponed until the time of sale and because the holder of the bond has control over when these taxes are paid (tax timing option). In order to examine the effect of taxes, we group bonds by coupon and examined the model errors. Table 5 shows the results for Baa rated industrial bonds. The results for other ratings are similar. The entries in Panel B represent model prices minus invoice price across six coupon categories and different maturities. Panel A shows the number of bond months in each category.

If taxes matter, we would expect to see a particular pattern in this table. Spot rates capture average tax rates. High coupon bonds are tax disadvantaged. If taxes matter, their model price will be too high and we will observe a positive pricing error. This is what we see in Table 5. In addition, as shown in Table 5, the longer the maturity of the bond, the more significant the pricing error becomes. For bonds with coupons below the average coupon in a risk class we should get the opposite sign (a negative sign) on the pricing error and the size of the error should become more negative with the maturity of the bond. This is the pattern shown in Table 5. Using a two-way analysis of variance test we can reject at the 1% level the hypothesis that the average errors are unaffected by coupon and maturity. If we compared the overall pricing error for high

Table 5
Errors for industrial Baa bonds sorted by coupon and maturity

	[1,2) years	[2,4) years	[4,6) years	[6,8) years	[8,10) years	[10,11) years
Panel A: Number of bonds						
[0,5)%	57	58	0	0	0	0
[5,6.5)%	112	279	156	84	190	1
[6.5,8)%	144	501	584	774	1562	115
[8,9.5)%	470	1200	1185	1149	1273	125
[9.5,11)%	258	624	954	853	722	103
[11,15)%	69	179	116	70	12	2
Panel B: Average errors						
[0,5)%	−0.4363	−0.6707				
[5,6.5)%	−0.0381	−0.5762	−1.1603	−0.9723	−1.3549	−1.4769
[6.5,8)%	−0.0575	0.2403	−0.1202	−0.1021	−0.3126	−0.2746
[8,9.5)%	0.0497	0.0646	−0.0820	−0.0968	0.0789	−0.6200
[9.5,11)%	−0.0937	−0.0415	0.0991	0.4165	1.0066	0.6395
[11,15)%	0.2479	0.4590	0.7475	1.5713	2.5329	2.4079
Weighted average	−0.0190	0.0192	−0.0558	0.0660	0.0298	−0.1153

Panel B of this table shows the errors from discounting the promised payments for Baa rated bonds of industrial category. The errors are model prices minus the invoice prices. The columns are different maturity ranges and the rows are different coupon ranges. Panel A shows the number of bonds over which the averaging was done in each cell. Units are dollars per $100. The two null hypotheses are, (i) H_{01}: Average errors are the same across the coupon groups and (ii) H_{02}: Average errors are the same across the maturity groups. These hypotheses were tested by a two-way analysis of variance. The value of the F statistic for H_{01} and H_{02} are 122.49 and 6.35 respectively, implying the rejection of both the nulls at 1% level of significance.

coupon (11%, 15%) to low coupon bonds (5%, 6.5%), the difference is significant at the 1% level. [11]

4.4. Different recovery rates

The fourth reason investors might rate bonds differently within a risk class is because of different expectations about recovery. Firms go bankrupt, not individual bonds. Bond ratings are a combination of default probability and expected recovery rate. Since all bonds issued by one firm have the same default probability, bonds of the same firm with different ratings imply that the rating agency believes they will have different expected recovery rates (possibly due to different seniority). Thus investors should realize that Moody's believes that an A bond of an Aa firm has a different expected recovery rate than an Aa bond of the same firm.

If investors place the same weight on default probability and recovery rate as Moody's, then sorting a bond rating class by difference in company ratings and not bond ratings should not result in pricing errors being related to the company rating. Examining Table 6 shows that bonds where the bond rating is higher than the

---

[11] It is always possible that coupon is in part a proxy for an effect other than taxes.

Table 6
Model errors due to differences in bond and company rating

	Financial sector			Industrial sector		
	Aa	A	Baa	Aa	A	Baa
Panel A: Number of pricing error observations						
Higher	3385	1737	145	1211	4355	1108
Same	5086	19 261	1839	3420	14 201	9537
Lower	2	11 396	8344	0	888	2604
Panel B: Average error						
Higher	0.006	0.588	0.887	0.306	0.147	0.854
Same	−0.040	−0.025	0.427	−0.168	−0.027	0.093
Lower	−0.097	−0.105	−0.135	–	−0.615	−0.866
t-scores between higher and lower	0.860	23.901	7.796	10.518	25.700	34.884

Each risk class is separated into three groups, one in which the bond is rated higher than the issuing company, one in which the bond is rated lower than the issuing company, and one in which the bond and the issuing company are equally rated. Panel A gives the number of bond price observations for each group of bonds. Panel B gives the average error, defined as model price minus invoice price. Units are dollars per $100 bond.

company rating have model prices above invoice prices. When the model price is above the invoice price, investors are requiring a higher rate of return in pricing the bond. For example, bonds that are rated AA in a company rated A have higher expected recovery rates than bonds rated A in a company rated A. Since, from Table 6, investors price these bonds lower, investors are placing more weight on bankruptcy probability and less on estimated recovery rates than Moody's does. The same logic holds for bonds ranked below the company rating. The differences are highly significant except for AA financials.

This raises another question. Could pricing be improved by discounting bonds at spot rates estimated from groups formed by using company rating rather than formed by bond rating? When we use company ratings to form groups and estimate spots the pricing errors are much larger. Bonds should be priced from discount rates estimated from groups using bond rating. However, taking into account the difference between bond rating and company rating adds information.

4.5. Bond age

We explore one other reason why bonds in a particular rating class might be viewed differently by investors: age of the bond. While the finance literature presents no economic reason why this might be true except for liquidity effects with new issues, it is a common way to present data in the corporate bond area, and it is an important consideration if one wants to model rating drift as a Markov process. [12]

---

[12] For example, Moody's typically presents data on the default rates as a function of the age of the bonds.

The issue is whether a bond with 15 years to maturity rated A, and 10 years old, is different from a bond with the same characteristics but two years old. When we examined this issue, except for new issues, there was no age effect. Thus there is no definitive evidence that the Markov assumption is being violated, and no definitive evidence that age of the bond is an important characteristic for classification. We believe that the new bond effect (age under one year) shows up because it represents a liquidity effect. New bonds tend to be more liquid during their first year of existence.

5. Adjusting for differences

We have now shown that a number of factors combined one at a time cause bonds within the same Moody's classification to have systematic price differences. The next step is to examine what proportion of the variation in errors across bonds can be explained by these factors and whether they are important when considered jointly. In addition, we do more formal statistical testing in this section.

Our prior analysis has shown the following influences are important:

1. A plus or minus rating within each risk letter classification. Furthermore, the importance is a function of maturity.
2. Differences in S&P and Moody's rating.
3. The coupon on a bond.
4. Differences in bond and company ratings.
5. Issued within the past one year.

To estimate the adjustment function we regressed model errors on a series of variables to capture simultaneously the impact of the influences discussed above. The variables are discrete except for coupon which is continuous. The regression we estimated is

$$E_j = \alpha + \sum_{i=1}^{8} B_i V_{ij} + e_j, \tag{3}$$

where E_j = the error measured as model price minus invoice price for bond j; V_{1j} = the maturity of a bond if it is rated plus otherwise zero; V_{2j} = the maturity of a bond if it is rated minus, otherwise zero; V_{3j} = dummy variable which is 1 if S&P rates a bond higher than Moody's, otherwise zero; V_{4j} = dummy variable which is 1 if Moody's rates a bond higher than S&P, otherwise zero; V_{5j} = the coupon on the bond minus the average coupon across all bonds; [13] V_{6j} = dummy variable which is 1 if the company has a higher rating than the bond, otherwise zero; V_{7j} = a dummy

---

[13] This variable was demeaned as not to transfer the average tax effect to the intercept.

variable which is 1 if the bond has a higher rating than the company, otherwise zero; V_{8j} = a dummy variable which is 1 if the bond is less than 1 year of age, otherwise zero; B_i = the sensitivity of errors to variable i.

The regression is estimated for bonds within each rating class for industrials and financials separately. Results are shown in Table 7. Almost all regression coefficients are statistically significant at the 1% level in every sample and have the sign that we would expect to see. The adjusted R^2 vary between 0.05 and 0.3 and average 0.18.

If we examine the regression coefficients one at a time we see very strong results. For plus rating the regression has the right sign for all rating categories and five of the six coefficients are significant at the 1% level. For minus ratings the coefficient has the right sign and is significant for five of the six categories. In the one group where the sign is inconsistent with what we would expect the coefficient is both small and not statistically significantly different from zero at the 5% level. When interpreting the signs, recall that plus-rated bonds are expected to have a negative error since the model price overestimates their risk.

Turning to bonds which have a S&P rating different from their Moody's rating, we find that the S&P rating contains added information about prices. For differences in ratings in either direction, the coefficient has the appropriate sign in all cases and is significantly different from zero at the 1% level in all but one case.

We have hypothesized that high coupon bonds were less desirable. The coupon variable has the correct sign in all cases and a coefficient which is significantly different from zero (at the 1% level) in five of the six case. While we reasoned that the impact of company and bond ratings were ambiguous because it depends on the weight the investor places on recovery rate versus probability of bankruptcy, the results tell a very consistent story. All of the 10 groups examined had consistent signs (two did not have enough observations to estimate the coefficients). Furthermore, 7 had

Table 7
Coefficients and their significance from the regression of model errors on bond and company characteristics

Variable	Financial sector			Industrial sector		
	Aa	A	Baa	Aa	A	Baa
Intercept	−0.022*	−0.018*	0.423*	−0.093*	0.082*	−0.195*
Plus∗maturity	−0.008	−0.055*	−0.005*	−0.010*	−0.069*	−0.071*
Minus∗maturity	0.014*	0.061*	0.123*	−0.003	0.030*	0.159*
S&P > Moody's	−0.274*	−0.283*	−0.124*	−0.109*	−0.257*	−0.086*
Moody's > S&P	0.035**	0.147*	0.456*	0.333*	0.167*	0.982*
Coupon	0.051*	0.059*	0.071*	0.110*	0.101*	0.155*
Company > bond	–	−0.010	−0.570*	–	−0.222*	−0.407*
Bond > company	0.018	0.487*	0.183	0.379*	0.075*	0.686*
Age < 1.0	−0.135*	−0.119*	−0.083*	−0.224*	−0.155*	−0.210*
Adjusted R^2	0.053	0.219	0.109	0.182	0.184	0.325

All variables are zero one variables except coupon which is the bonds coupon rate and plus times maturity and minus times maturity which are zero one variables times maturity.
*Coefficient is different from zero at the 1% level of significance. **5% level of significance.

coefficients which were statistically significantly different from zero at the 1% level. These results indicate that investors place more emphasis on bankruptcy risk than the relative weight it is given in bond ratings. Finally, new bonds sell at a premium. All the estimates have the right sign and are statistically different from zero at the 1% level.

The next logical step would be to take the influences discussed above into account in defining new classifications (homogeneous groups) of bonds that exist within each Moody's risk classification and to estimate new spot curves within each classification. Unfortunately, this would result in such fine classifications that we would have too few bonds within each classification to estimate spot curves with any accuracy.

An alternative is to introduce these variables directly into the procedure for estimating spot curves so that the spot rates determined for any bond are not only a function of the Moody's risk class to which the bond belongs, but the rates are conditional on all of the variables we have found important in the previous section. The spot rates developed from this procedure can then be used to price bonds and the resulting model prices compared with model prices arrived at only using Moody's ratings.

We modify the Nelson–Siegel estimation approach to take added influences into account. Because of the number of influences we found important and the number of parameters, as well as ratios and cross products of parameters in the Nelson–Siegel procedure we needed to make some simplifying assumptions about the nature of changes in the term structure caused by adding these influences. We assumed that each of the variables discussed in the previous section of this paper could effect the level but not the shape of the corporate term structure. For example, our estimation procedure assumes that the Baa+ and Baa− spot term structure curves are parallel to each other and the Baa spot term structure curve. To the extent that this simplification of the effect of variables is inappropriate it will bias our results against attributing importance to the influences we examine.

The new equation used to estimate the term structure for any bond with a particular Moody's rating is found by using the following modification of Eq. (3):

$$r_{0t} = a_0 + (a_1 + a_2)\left[\frac{1 - e^{-a_3 \cdot t}}{a_3 t}\right] - a_2 e^{-a_3 t} + \sum_{j=1} b_i V_{ij}, \qquad (4)$$

where

$$P_{i0} = \sum_{t=1}^{T} D_t CF_{it},$$

$$D_t = e^{-r_{0t} t}.$$

This equation was estimated within each Moody's risk class for industrial and financial bonds separately. This allowed us to estimate a spot curve for any bond and to arrive at a model price based on these spots.

Table 8
Coefficients and the significance of the variables added to the Nelson and Siegel (1987) equation for estimating spots

Variable	Financial sector			Industrial sector		
	Aa	A	Baa	Aa	A	Baa
Plus * maturity	0.004*	−0.011*	−0.018*	−0.029*	−0.008*	−0.015*
Minus * maturity	−0.008*	0.019*	0.032*	−0.013*	0.012*	0.037*
S&P > Moody's	−0.104*	−0.046*	−0.038	0.023	−0.067*	−0.101*
Moody's > S&P	0.029*	0.070*	0.289*	0.089*	0.119*	0.433*
Coupon	0.602*	0.440*	0.165**	0.671*	0.352*	0.602*
Company > bond	0.017	0.011	0.322*	0.185*	−0.025*	0.045*
Bond > company	−0.051*	−0.026*	−0.024	0.067*	−0.018*	−0.006*
Age < 1.0	−0.050*	0.007	0.157*	0.115*	−0.084*	−0.006*

All variables are zero one variables except coupon which is the bonds coupon rate and plus times maturity and minus times maturity which are zero one variables times maturity.
*Coefficient is different from zero at the 1% level of significance. **5% level of significance.

The results of this analysis are shown in Tables 8 and 9. [14] Before examining the improvement in pricing errors due to inclusion of our set of additional variables into the procedure used to estimate spot rates (Eq. (4)), let us examine the sign and statistical significance of the coefficient on each variable. The numbers reported in Table 8 are the mean value for each coefficient over the 120 months as well as the significance of the difference of the mean from zero. Examining the effect of Moody's plus and minus ratings on estimates of the yield curve we see that 10 of the 12 coefficients (across three rating classes, two sectors, and two types) have the expected sign and are highly significant. In one case where the sign is wrong the coefficient is insignificant. The two variables measuring differences between Moody's and S&P ratings have the hypothesized sign in 11 of 12 cases (10 significant) and in the one case where it has the wrong sign, it is insignificant. Coupon as expected is significantly positive in all six cases. The results for age and the difference between company rating and bond rating are more mixed. For company rating different than bond rating in 12 of the 12 cases, the coefficients have the right sign (6 significant), but in two of the cases where the signs are wrong, the coefficients are significant. Finally, for age, the coefficients have the wrong sign and are significant as often as they have the right sign and are significant.

In column c, Table 9 we show the average absolute errors from using Eq. (4) to value Baa, A and Aa rated bonds for industrial and financial companies for two five-year periods and the overall 10-year period. The average absolute error varies from 33 cents per 100 bond for the financial Aa category up to 90 cents for the industrial Baa rated category. How can we judge the improvement from incorporating these additional factors? One way is to compare these errors (column c) with the errors (column a) when rating alone is accepted as a metric for homogeneous risk.

---

[14] The numbers here are somewhat different from those shown in Table 2 because the sample is changed since we require information on all variables rather than just price.

Table 9
Reduction in pricing errors by incorporating the bond characteristics information

Risk category	Unadjusted errors (a)	Errors adjusted by previous months' errors (time series adjustment) (b)	Errors adjusted by bond characteristics (characteristics adjustment) (c)	Fraction of error reduction obtained by characteristics adjustment $d = (a - c)/(a - b)$
Panel A: Mean absolute pricing errors over the full time period (1/1987 to 12/1996)				
Financial Aa	0.335	0.237	0.332	2.18%
Financial A	0.593	0.374	0.495	44.86%
Financial Baa	0.884	0.575	0.794	29.16%
Industrial Aa	0.475	0.297	0.407	37.97%
Industrial A	0.625	0.380	0.525	40.71%
Industrial Baa	1.172	0.613	0.898	48.96%
Panel B: Mean absolute pricing errors over the first half time period (1/1987 to 12/1991)				
Financial Aa	0.336	0.252	0.374	−44.59%*
Financial A	0.708	0.481	0.589	52.50%
Financial Baa	0.943	0.716	0.880	27.60%
Industrial Aa	0.505	0.336	0.411	55.98%
Industrial A	0.679	0.440	0.577	42.86%
Industrial Baa	1.147	0.701	0.868	62.65%
Panel C: Mean absolute pricing errors across the second half time period (1/1992 to 12/1996)				
Financial Aa	0.333	0.221	0.291	37.44%
Financial A	0.479	0.268	0.402	36.61%
Financial Baa	0.826	0.435	0.708	30.06%
Industrial Aa	0.445	0.258	0.404	21.73%
Industrial A	0.570	0.319	0.473	38.65%
Industrial Baa	1.197	0.525	0.929	39.90%

This table shows the extent of improvement that can be achieved in the model prices of bonds by using the information on bond characteristics. The pricing error is defined as $(P_{mi} - P_{ai})$, where P_{mi} is the model price of bond i obtained by discounting its cash flows by spot rates derived from spline fitting and P_{ai} is the actual invoice price of this bond. Column (a) shows the mean absolute pricing error obtained when the model did not use the information on the bond characteristics. Mean absolute pricing error for a given risk category (e.g. industrial BBB) was obtained by averaging the absolute pricing errors of all the bonds in that risk category across all the months in the time period mentioned. Column (b) shows the mean absolute pricing error when the model price of each bond was adjusted by subtracting back the average of last six months' pricing errors on that bond (time series adjustment). Column (c) shows the mean absolute pricing errors obtained when the model incorporated the information about the bond characteristics (characteristics adjustment). Column (d) shows the reduction in mean absolute errors obtained by characteristics adjustment as a fraction of the reduction obtained through time series adjustment.
*In this case, the characteristics adjustment led to a bigger error rather than a reduction in error.

In each of the six categories for the 10-year period and for 11 of the 12 five-year categories the error has been reduced. In each of these 11 cases, the reduction in model error is statistically significant at the 1% level. [15] Note that since the spot rates are estimated each month, and since all the information is known at that time, the improvement in model pricing errors is attainable by researchers or investors.

---

[15] The t's associated with the differences in errors average 5.1–17.67 with the typical one about 10.

We wish to get a better measure of the improvement in estimates of the spot yield curve with our added set of variables. When we only employ risk class as a measure of homogeneity pricing errors will tend to persist over time for three reasons: (1) because the additional qualities of a bond not captured by risk class would be expected to impact the price and since these qualities change slowly over time, if at all, we should observe persistence, (2) firm effects may be present and (3) dealer prices may be sticky since dealers may not correct their misestimation quickly over time.

One way to correct for all three of these reasons is to adjust the price predicted for a bond by past errors in pricing the bond. [16] These results are shown in column b of Table 9. To measure this we used the average of the last six months' errors. Table 9 shows that introducing past errors in the analysis reduces the error based on Moody's ratings by a significant amount. For example, for Baa industrial bonds the size of the average absolute error is reduced from $1.17 per $100 bond to $0.61. Recall that this reduction occurred because of omitting bond characteristics which should have been included in estimating bond spot rates, firm affects, and/or stickiness in dealer prices. We now estimate what percentage of this reduction is just due to omitting the set of bond characteristics we have been examining (Eq. (4)). This is shown in column d of Table 9. For industrial bonds incorporating our set of fundamental characteristics into the estimates of spot rates accounts for a decrease of between 38% and 49% of the aggregate impact of the three influences discussed above. We have not been quite as successful for financial bonds but we have reduced the error by 2–45%. This analysis shows that the set of variables we have examined are important influences in determining the risk structure of corporate bonds and capture a significant portion of the influences that affect bond prices beyond that captured by rating class.

6. Conclusion

In this paper we explore the characteristics of corporate bonds that effect their price. All rating-based techniques involve working with a homogeneous population of bonds. The common assumption is that Moody's or Standard & Poors ratings are a sufficient metric for homogenity. We expand previous analysis to discover what characteristics of bonds are priced differently by the market. We find that several characteristics of bonds and bond rating beyond the simple rating categories of Moody's and Standard and Poor convey information about the pricing of corporate bonds. In particular the following five influences are important:

1. The finer rating categories introduced by both rating agencies when combined with maturity measures.
2. Differences between S&P and Moody's ratings.

---

[16] The model price is reduced (increased) by the amount that the model price overestimated (underestimated) the bonds actual price.

3. Differences in the rating of a bond and the rating of the company which issued that bond.
4. The coupon on the bond.
5. Whether a bond is new and has traded for more than one year.

We adjust for these characteristics and show the improvement in pricing error and spot yield curve estimation. Bond pricing models which are based on ratings whether the models involve discounting cash flows or employing risk-neutral probabilities need to be adjusted for these influences. Failure to do so has resulted in the development of bond pricing models which are not only less efficient but may also be biased with respect to important classes of bonds.

References

Billingsley, R.S., Lamy, R., Marr, E., Thompson, R., 1985. Split ratings and bond reoffering yields. Financial Management 14, 59–65.

Boardman, C.M., McEnally, R.W., 1981. Factors affecting seasoned corporate bond prices. Journal of Financial and Quantitative Analysis 16 (2), 207–226.

Bradley, F., 1991. Neglected factors in the market pricing of eurodollar bonds. Journal of Portfolio Management 17 (2), 62–73.

Cumby, R., Evans, M., 1997. The Term Structure of Credit Risk: Estimates and Specification Tests. Georgetown University.

Dahlquist, M., Svensson, L., 1996. Estimating the term structure of interest rates for monetary policy analysis. Scandinavian Journal of Economics 98, 163–181.

Das, S., Tufano, P., 1996. Pricing credit sensitive debt when interest rates and credit spreads are stochastic. Journal of Financial Engineering 5, 161–198.

Duffee, G.R., 1999. Estimating the price of default risk. Review of Financial Studies 12, 197–226.

Duffie, D., Singleton, K.J., 1997. Modeling term structures of defaultable bonds. Review of Financial Studies 12 (4), 687–720.

Elton, E.J., Gruber, M., Agrawal, D., Mann, C., 2001. Explaining the rate spread on corporate bonds. Journal of Finance 56 (1), 247–261.

Green, R., Odegaard, B., 1997. Are there tax effects in the relative pricing of U.S. government bonds? Journal of Finance 2, 609–633.

Jaffee, D., 1975. Cyclical variations in the risk structure of interest rates. Journal of Monetary Economics 1, 309–325.

Jarrow, R., Lando, D., Turnbull, S., 1997. A Markov model for the term structure of credit spreads. Review of Financial Studies 10, 481–523.

Jones, E., Mason, S., Rosenfeld, E., 1984. Contingent claims analysis of corporate capital structures: An empirical investigation. The Journal of Finance 39, 611–625.

Lamy, R., Thompson, G.R., 1988. Risk premia and the pricing of primary issue bonds. Journal of Banking and Finance 12 (2), 585–601.

Lando, D., 1997. Modeling bonds and derivatives with default risk. In: Dempster, M., Pliska, S. (Eds.), Mathematics of Derivative Securities. Cambridge University Press, pp. 369–393.

Merrick, J., 1999. Crisis dynamics of implied default recovery rates: Evidence from Russia and Argentina. New York University Working Paper Series.

Madan, D., Unal, H., 1998. Pricing the risks of default. Review of Derivatives Research 2, 121–160.

Merton, R., 1974. On the pricing of corporate debt: The risk structure of interest rates. The Journal of Finance 29, 449–470.

Nelson, R., Siegel, F., 1987. Parsimonious modeling of yield curves. The Journal of Business 60, 473–489.

Perry, L.G., Liu, P., Evans, D.A., 1988. Modified bond ratings: Further evidence on the effect of split ratings on corporate bond yields. Journal of Business Finance and Accounting 15, 231–241.

Warga, A., 1998. Fixed Income Database. University of Houston, Houston, TX.